PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA MECÂNICA

UNIVERSIDADE FEDERAL DE MINAS GERAIS

Escola de Engenharia

Prova de seleção de candidatos 2025/2

Orientações - LEITURA OBRIGATÓRIA:

- Esta prova terá duração de 120 minutos, contados a partir do horário de início de sua realização;
- A prova é composta por 20 questões de múltipla escolha com 5 opções (A, B, C, D e E);
- O candidato deve marcar uma opção por questão, somente;
- É obrigatório o preenchimento da última folha, intitulada **FOLHA DE RESPOSTAS**;
- A correção da prova será feita com base no que estiver apresentado na **FOLHA DE RESPOSTAS**, nada mais;
- Não é permitido desgrampear a prova, sob pena de desclassificação;
- Ao final da prova, é obrigatória a devolução de todas as páginas desta prova;
- É obrigatória a assinatura nesta página, confirmando a leitura destas orientações, e a concordância com o disposto;
- Todo o conteúdo da prova está em **língua inglesa**.

Assinatura do(a) candidato(a): _____

Question 1. Evaluate the limit:

$$\lim_{x \to -6} \frac{\sqrt{10 - x} - 4}{x + 6}$$

(A) $\frac{1}{8}$

(B) $-\frac{1}{8}$

(C) 0

(D) 4

(E) The limit does not exist

Question 2. Let the function $f(x)$ be defined as

$$f(x) = \begin{cases} \frac{x+2}{2}, & \text{if } x \leq 3 \\ \frac{12-2x}{3}, & \text{if } x > 3 \end{cases}$$

Evaluate the limit: $\lim_{x \to 3^-} f(x)$

(A) 3

(B) $\frac{8}{3}$

(C) $\frac{5}{2}$

(D) 4

(E) The limit does not exist

Question 3. Let

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x \leq 2 \\ 4x - 3, & \text{if } x > 2 \end{cases}$$

Is the function $f(x)$ differentiable at $x = 2$?

(A) No, because the function is not continuous at $x = 2$

(B) No, because the left and right derivatives are not equal

(C) No, because the left and right limits are not equal

(D) Yes, because the function is continuous but not differentiable at $x = 2$

(E) Yes, because the function is continuous and the left and right derivatives agree at $x = 2$

Question 4. Let

$$f(x) = \begin{cases} ax^3, & \text{if } x \leq 2 \\ x^2 + b, & \text{if } x > 2 \end{cases}$$

Find the values of a and b such that the function $f(x)$ is differentiable everywhere.

(A) $a = \frac{1}{2}, \quad b = -2$

(B) $a = \frac{3}{4}, \quad b = -1$

(C) $a = 1, \quad b = 0$

(D) $a = \frac{2}{3}, \quad b = -\frac{1}{3}$

(E) None of the above

Question 5. Let $f(x) = -x + \tan x$. Find $f'(x)$.

(A) $-1 + \sec^2 x$

(B) $-1 + \tan x$

(C) The correct derivative is not listed

(D) $-\sec^2 x$

(E) $1 + \sec^2 x$

Question 6. Let

$$y = \frac{3(1 - \sin x)}{2 \cos x}$$

Find $\frac{dy}{dx}$.

(A) $\frac{-3 \cos x}{2 \cos x}$

(B) $\frac{3 \sin x}{2 \cos x}$

(C) $\frac{3}{2} \sec x(\tan x - \sec x)$

(D) Result not listed

(E) $\frac{3 \cos x(1+\sin x)}{4 \cos^2 x}$

Question 7. Let $f(\theta) = \tan^2(5\theta)$. Find $f'(\theta)$.

(A) $2\tan(5\theta)\sec^2(\theta)$

(B) $10\tan(5\theta)\sec^2(5\theta)$

(C) $5\tan(5\theta)\sec^2(5\theta)$

(D) $2\sec^2(5\theta)$

(E) None of the above

Question 8. A 15-centimeter pendulum swings according to the angular equation

$$\theta(t) = 0.2\cos(8t),$$

where $\theta(t)$ is the angular displacement (in radians) and t is the time (in seconds). What is the maximum angular displacement, and what is the rate of change of θ at $t = 3$ s?

(A) 0.2 rad, $\frac{d\theta}{dt}(3) = -1.45$ rad/s

(B) 8 rad, $\frac{d\theta}{dt}(3) = -1.45$ rad/s

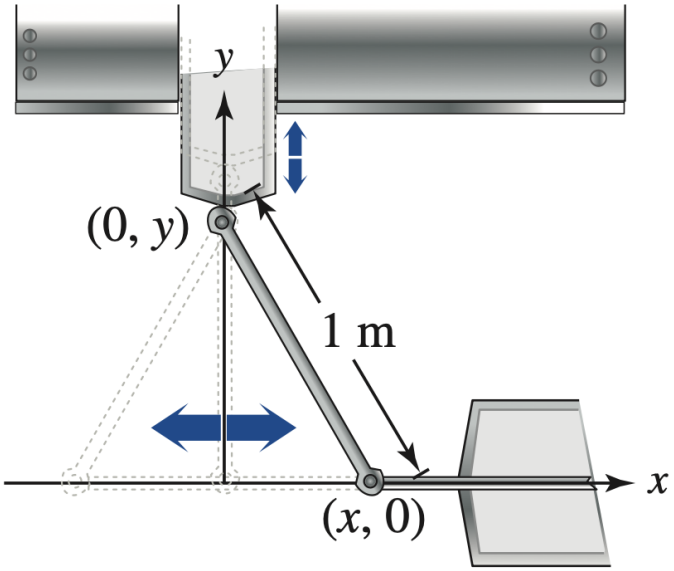
(C) 0.2 rad, $\frac{d\theta}{dt}(3) = 1.45$ rad/s

(D) 0.2 rad, $\frac{d\theta}{dt}(3) = -0.90$ rad/s

(E) 0.2 rad, $\frac{d\theta}{dt}(3) = 0$ rad/s

Question 9. A movable rod of length 1 meter has endpoints at coordinates $(x, 0)$ and $(0, y)$, as shown below. The position of the x-end is given by:

$$x(t) = \frac{1}{2}\sin\left(\frac{\pi t}{6}\right)$$



Find the time required for one complete cycle of the rod's motion.

(A) 6 seconds

(B) π seconds

(C) 12 seconds

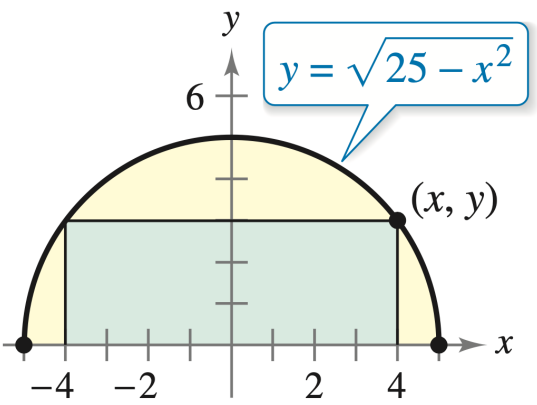
(D) $\frac{6}{\pi}$ seconds

(E) The correct answer is not listed

Question 10. A rectangle is inscribed under the semicircle defined by

$$y = \sqrt{25 - x^2}$$

with its base lying along the x-axis and top corners touching the semicircle. What are the dimensions of the rectangle (i.e., base along the x-axis and height) that maximize its area?



(A) Height = 5, Base = 5

(B) Height = $\frac{5\sqrt{2}}{2}$, Base = $5\sqrt{2}$

(C) Height = $\frac{10}{\sqrt{2}}$, Base = $\frac{5}{\sqrt{2}}$

(D) Height = $\frac{5}{\sqrt{2}}$, Base = $\frac{10}{\sqrt{2}}$

(E) None of the options above are correct

Question 11. The escape velocity from Earth's gravity is obtained by solving:

$$\int v\,dv = -GM \int \frac{1}{y^2}\,dy$$

where v is the velocity, y is the distance from the center of the Earth, G is the gravitational constant, and M is the mass of Earth.

Which of the following correctly relates v and y, assuming an initial velocity v_0 at a radius $y = R$?

(A) $v^2 = v_0^2 + GM\left(\frac{1}{y^2} - \frac{1}{R^2}\right)$

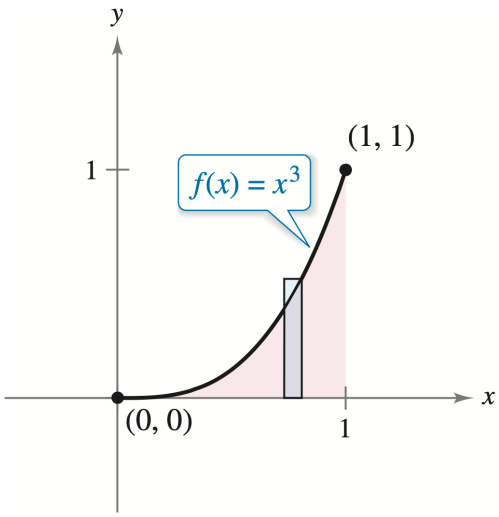
(B) $v^2 = v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)$

(C) $v = v_0 + 2GM\left(\frac{1}{R} - \frac{1}{y}\right)$

(D) $v = \sqrt{2GM}\left(\frac{1}{y} + \frac{1}{R}\right)$

(E) The correct expression is not listed

Question 12. Consider the function $f(x) = x^3$ on the interval $[0, 1]$, as shown in the figure below. What is the area under the curve between $x = 0$ and $x = 1$?



(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

(E) The area cannot be determined

Question 13. Analyze the following statements:

I. $\displaystyle\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

II. If the size of the subintervals in a partition gets smaller and smaller, then the number of subintervals increases without bound.

III. The definite integral $\int_a^b f(x)\,dx$ is always positive.

Which of the statements above are true?

(A) Only I

(B) Only II and III

(C) Only I and II

(D) All three

(E) None of the above

Question 14. Evaluate the definite integral:

$$\int_{-1}^{1} x(x^2+1)^3\,dx$$

(A) 0

(B) $\frac{1}{4}$

(C) 1

(D) $\frac{2}{5}$

(E) The integral is undefined

Question 15. Determine which of the following statements are true.

I. $\displaystyle\int (2x+1)^2\,dx = \frac{1}{3}(2x+1)^3 + C$

II. $\displaystyle\int_{-10}^{10} (ax^3+bx^2+cx+d)\,dx = 2\int_{0}^{10} (bx^2+d)\,dx$

III. $\displaystyle 4\int \sin x \cos x\,dx = -\cos 2x + C$

Which of the above statements are true?

(A) Only I

(B) Only II and III

(C) Only I and III

(D) All three

(E) None of the above

Question 16. Compute the indefinite integral:

$$\int \frac{x^2-4}{x}\,dx$$

(A) $\frac{x^2}{2} + 4\ln|x| + C$

(B) $\frac{x^2}{2} - 4\ln|x| + C$

(C) $x^2 - \frac{4}{x} + C$

(D) $\ln(x^2-4) + C$

(E) None of the above

Question 17. Evaluate the definite integral:

$$\int_{-1}^{2} 2^x \, dx$$

(A) $\frac{3}{\ln 2}$

(B) $\frac{7}{2 \ln 2}$

(C) $\frac{4}{\ln 2}$

(D) $\frac{5}{\ln 2}$

(E) None of the above

Question 18. Given that the general solution of the differential equation $y' + 2y = 0$ is $y = Ce^{-2x}$, and that $y = 3$ when $x = 0$, find the particular solution.

(A) $y = 3e^{2x}$

(B) $y = 3e^{-2x}$

(C) $y = 2e^{-3x}$

(D) $y = e^{-6x} + 3$

(E) None of the above

Question 19. Consider the differential equation:

$$xy' - 2y = x^3 e^x$$

Is the function $y = x^2 e^x$ a solution of the equation?

(A) Yes, because it satisfies the equation for all x

(B) Yes, but only when $x = 0$

(C) No, because it leads to a contradiction

(D) No, because the derivative of y is not continuous

(E) Cannot be determined without initial conditions

Question 20. Consider the parametric equations:

$$x = 4t, \quad y = 3t - 2$$

Evaluate the slope $\frac{dy}{dx}$ and the concavity $\frac{d^2y}{dx^2}$ at $t = 3$.

(A) $\frac{dy}{dx} = \frac{3}{4}, \quad \frac{d^2y}{dx^2} = 0$

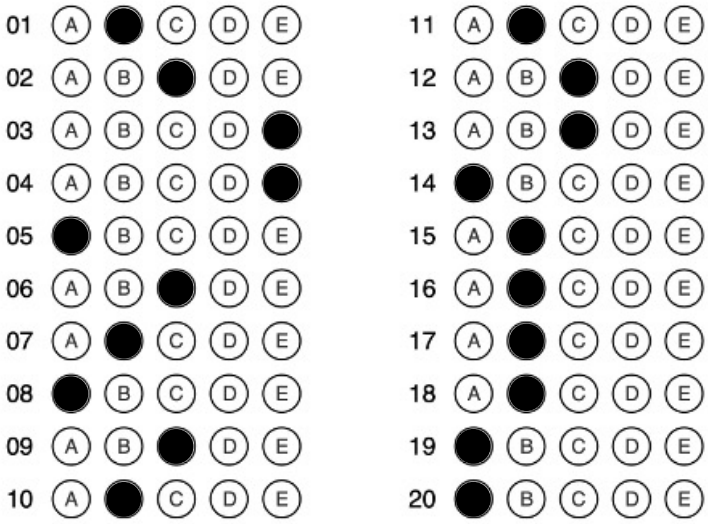
(B) $\frac{dy}{dx} = \frac{4}{3}, \quad \frac{d^2y}{dx^2} = 0$

(C) $\frac{dy}{dx} = 3, \quad \frac{d^2y}{dx^2} = 4$

(D) $\frac{dy}{dx} = \frac{3}{4}, \quad \frac{d^2y}{dx^2} = 1$

(E) Cannot be determined without x in terms of y

FOLHA DE RESPOSTAS



Instruções:

- Preencha apenas uma opção por questão.
- Use caneta azul ou preta.
- Não amasse, dobre ou rasure esta folha.
- A correção da prova será feita exclusivamente com base nesta folha.

Nome: _____

Assinatura do Candidato: _____