

PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA MECÂNICA

UNIVERSIDADE FEDERAL DE MINAS GERAIS

Escola de Engenharia

Prova de seleção de candidatos 2026/1

Orientações - LEITURA OBRIGATÓRIA:

- Esta prova terá duração de 120 minutos, contados a partir do horário de início de sua realização;
- A prova é composta por 20 questões de múltipla escolha com 5 opções (A, B, C, D e E);
- O candidato deve marcar uma opção por questão, somente;
- É obrigatório o preenchimento da última folha, intitulada **FOLHA DE RESPOSTAS**;
- A correção da prova será feita com base no que estiver apresentado na **FOLHA DE RESPOSTAS**, nada mais;
- Não é permitido desgrampear a prova, sob pena de desclassificação;
- Ao final da prova, é obrigatória a devolução de todas as páginas desta prova;
- É obrigatória a assinatura nesta página, confirmando a leitura destas orientações, e a concordância com o disposto;
- Todo o conteúdo da prova está em **língua inglesa**.

Assinatura do(a) candidato(a): _____

Question 1. Find the limit:

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}.$$

- (A) 1
- (B) None of the others
- (C) 3
- (D) 4
- (E) 2

Question 2. Find the limit:

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}.$$

- (A) 5
- (B) None of the others
- (C) -5
- (D) -3
- (E) 0

Question 3. Find the limit:

$$\lim_{x \rightarrow 2} f(x), \quad \text{where} \quad f(x) = \begin{cases} (x-2)^2, & x \leq 2, \\ 2-x, & x > 2. \end{cases}$$

- (A) 0
- (B) -1
- (C) 1
- (D) None of the others
- (E) 2

Question 4. Evaluate the limit:

$$\lim_{x \rightarrow -1^+} \frac{x+1}{x^3+1}.$$

- (A) $\frac{1}{3}$
- (B) 0
- (C) None of the others
- (D) $+\infty$
- (E) 1

Question 5. Find the derivative of the function:

$$f(x) = 6\sqrt{x} + 5 \cos x$$

- (A) None of the others
- (B) $f'(x) = 6x^2 - 5 \sin x$
- (C) $f'(x) = 3x^{1/2} + 5 \sin x$
- (D) $f'(x) = \frac{3}{\sqrt{x}} - 5 \sin x$
- (E) $f'(x) = \frac{6}{\sqrt{x}} + 5 \cos x$

Question 6. Let $y(x) = \frac{1}{x}$ for $x > 0$. Evaluate the left-hand side of the differential equation

$$x^3 y'' + 2x^2 y' = ?$$

- (A) None of the others
- (B) 2
- (C) 0
- (D) -2
- (E) $\frac{1}{x}$

Question 7. Find the derivative of the function:

$$f(t) = \sin^3(4t)$$

(A) $f'(t) = 12 \sin(4t) \cos^2(4t)$

(B) $f'(t) = 3 \sin^3(4t) \cos(4t)$

(C) $f'(t) = 4 \sin^2(4t) \cos(4t)$

(D) None of the others

(E) $f'(t) = 12 \sin^2(4t) \cos(4t)$

Question 8. A rotating beacon is located 1 km off a straight shoreline. The beacon rotates at a rate of 3 revolutions per minute. How fast (in km/h) does the beam of light appear to move along the shoreline at a point $\frac{1}{2}$ km from the closest point to the beacon?

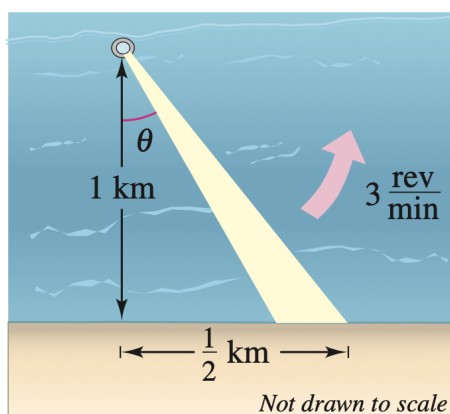


Figure 1: Winch and pipe system used in Question 8.

(A) 300π km/h

(B) 225π km/h

(C) 450π km/h

(D) None of the others

(E) 90π km/h

Question 9. Refer to Figure 2. A swimming pool is 12 m long, 6 m wide, 1 m deep at the shallow end, and 3 m deep at the deep end (floor slopes linearly). If there is currently 1 m of water at the deep end, what percent of the pool is filled?

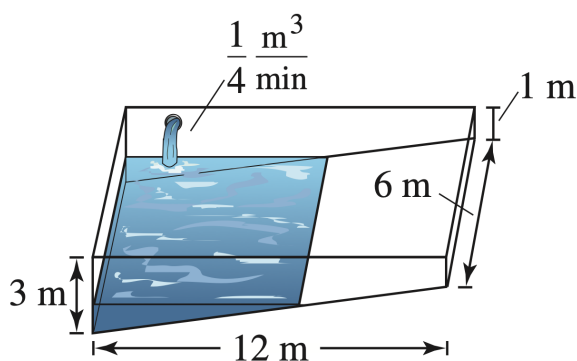


Figura 2: Questions 8 and 9.

- (A) 25%
- (B) 12.5%
- (C) 8.33%
- (D) 6.25%
- (E) None of the above

Question 10. Refer to Figure 2. Water is being pumped into the same pool at a rate of $\frac{1}{4} \frac{\text{m}^3}{\text{min}}$. At the moment when the depth at the deep end is 1 m, at what rate is the water level rising?

- (A) $\frac{1}{72} \frac{\text{m}}{\text{min}}$
- (B) None of the others
- (C) $\frac{1}{36} \frac{\text{m}}{\text{min}}$
- (D) $\frac{1}{144} \frac{\text{m}}{\text{min}}$
- (E) $\frac{1}{288} \frac{\text{m}}{\text{min}}$

Question 11. Consider the the function $y = \sqrt{x}$ on the interval $[0, 4]$. Which of the following values best approximates the definite integral

$$\int_0^4 \sqrt{x} \, dx ?$$

- (A) 5
- (B) -3
- (C) 10
- (D) 2
- (E) 8

Question 12. Consider the function

$$f(x) = \frac{1}{x - 4}.$$

Which of the following statements correctly describes the integrability of $f(x)$ on the interval $[3, 5]$?

- (A) The function is integrable on $[3, 5]$ because it is continuous on the entire interval.
- (B) The function is integrable on $[3, 5]$ because its only discontinuity is removable.
- (C) The function is not integrable on $[3, 5]$ because it has an infinite discontinuity at $x = 4$.
- (D) The function is integrable on $[3, 5]$ because improper integrals are always finite.
- (E) The function is integrable only on $[3, 4)$ and $(4, 5]$, but not on the closed interval $[3, 5]$.

Question 13. Evaluate the definite integral

$$\int_1^5 \frac{x}{\sqrt{2x-1}} dx.$$

- (A) 4
- (B) $\frac{16}{3}$
- (C) None of the others
- (D) $\frac{8}{3}$
- (E) 6

Question 14. Compute the indefinite integral

$$\int \frac{1}{\theta^2} \cos\left(\frac{1}{\theta}\right) d\theta.$$

- (A) $\sin\left(\frac{1}{\theta}\right) + C$
- (B) $-\cos\left(\frac{1}{\theta}\right) + C$
- (C) None of the others
- (D) $\cos\left(\frac{1}{\theta}\right) + C$
- (E) $-\sin\left(\frac{1}{\theta}\right) + C$

Question 15. Compute the indefinite integral

$$\int \frac{1}{4x - 1} dx.$$

- (A) $\ln |4x - 1| + C$
- (B) $\frac{1}{4} \ln |4x - 1| + C$
- (C) None of the others
- (D) $4 \ln |4x - 1| + C$
- (E) $-\frac{1}{4} \ln |4x - 1| + C$

Question 16. A circular region is given by the equation

$$(x - 2)^2 + y^2 = 1.$$

The region is revolved about the y -axis, forming a torus. Determine the volume of the resulting solid.

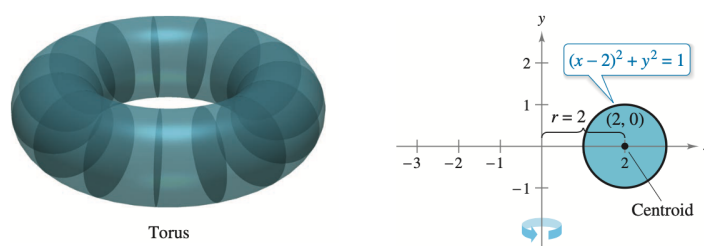
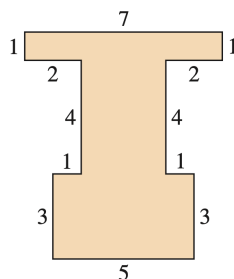


Figure 3: Finding Volume by the Theorem of Pappus.

- (A) $2\pi^2$
- (B) $4\pi^2$
- (C) $8\pi^2$
- (D) 16π
- (E) None of the others

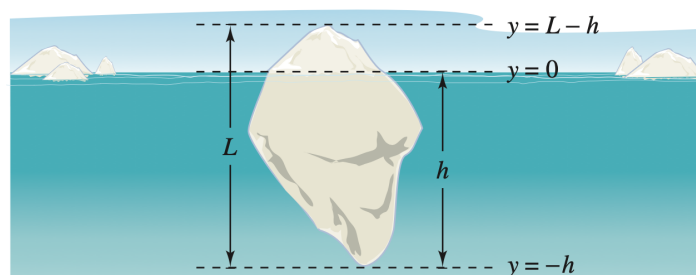
Question 17. A homogeneous planar lamina has the shape of an I-beam formed by three rectangles, as shown in the figure below. The bottom flange is a 5×3 rectangle, the central web is a 3×7 rectangle, and the top flange is a 7×1 rectangle. Each part is centered on the same vertical line.

Assume a coordinate system in which the origin is at the center of the bottom side of the lamina, the x-axis is horizontal, and the y-axis is vertical. What are the coordinates $(\bar{x}, \bar{y})$ of the center of mass of the lamina?



- (A) $\left(0, \frac{11}{2}\right)$
- (B) $\left(0, \frac{7}{2}\right)$
- (C) $\left(0, \frac{135}{34}\right)$
- (D) $\left(0, \frac{9}{2}\right)$
- (E) None of the others

Question 18. An iceberg floats in ocean water. The density of ocean water is $1.03 \times 10^3 \text{ kg/m}^3$, and the density of ice is $0.92 \times 10^3 \text{ kg/m}^3$. What percent of the total volume of the iceberg lies *below* the surface of the water?



- (A) $\approx 50\%$
- (B) $\approx 60\%$
- (C) $\approx 70\%$
- (D) $\approx 80\%$
- (E) $\approx 90\%$

Question 19. Compute the integral

$$\int x e^x dx.$$

- (A) $e^x(x + 1) + C$
- (B) $e^x(1 - x) + C$
- (C) $e^x(x - 1) + C$
- (D) $x e^x + C$
- (E) None of the others

Question 20. Compute the indefinite integral

$$\int \cos^4 x dx.$$

- (A) $\frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$
- (B) $\frac{x}{4} + \frac{\sin 2x}{8} + C$
- (C) $\frac{3x}{4} + \frac{\sin 2x}{2} + C$
- (D) $\frac{x}{2} + \frac{\sin 4x}{16} + C$
- (E) None of the others

FOLHA DE RESPOSTAS

01	☐ A	☐ B	☐ C	☐ D	☐ E	11	☐ A	☐ B	☐ C	☐ D	☐ E
02	☐ A	☐ B	☐ C	☐ D	☐ E	12	☐ A	☐ B	☐ C	☐ D	☐ E
03	☐ A	☐ B	☐ C	☐ D	☐ E	13	☐ A	☐ B	☐ C	☐ D	☐ E
04	☐ A	☐ B	☐ C	☐ D	☐ E	14	☐ A	☐ B	☐ C	☐ D	☐ E
05	☐ A	☐ B	☐ C	☐ D	☐ E	15	☐ A	☐ B	☐ C	☐ D	☐ E
06	☐ A	☐ B	☐ C	☐ D	☐ E	16	☐ A	☐ B	☐ C	☐ D	☐ E
07	☐ A	☐ B	☐ C	☐ D	☐ E	17	☐ A	☐ B	☐ C	☐ D	☐ E
08	☐ A	☐ B	☐ C	☐ D	☐ E	18	☐ A	☐ B	☐ C	☐ D	☐ E
09	☐ A	☐ B	☐ C	☐ D	☐ E	19	☐ A	☐ B	☐ C	☐ D	☐ E
10	☐ A	☐ B	☐ C	☐ D	☐ E	20	☐ A	☐ B	☐ C	☐ D	☐ E

Instruções:

- Preencha apenas uma opção por questão.
- Use caneta azul ou preta.
- Não amasse, dobre ou rasure esta folha.
- A correção da prova será feita exclusivamente com base nesta folha.

Nome: _____

Assinatura do Candidato: _____

FOLHA DE RESPOSTAS

01	☐ A	☐ B	☒ C	☐ D	☐ E
02	☐ A	☐ B	☒ C	☐ D	☐ E
03	☒ A	☐ B	☐ C	☐ D	☐ E
04	☒ A	☐ B	☐ C	☐ D	☐ E
05	☐ A	☐ B	☐ C	☒ D	☐ E
06	☐ A	☒ B	☐ C	☐ D	☐ E
07	☐ A	☐ B	☐ C	☐ D	☒ E
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09	☐ A	☒ B	☐ C	☐ D	☐ E
10	☐ A	☐ B	☐ C	☒ D	☐ E
11	☒ A	☐ B	☐ C	☐ D	☐ E
12	☐ A	☐ B	☒ C	☐ D	☐ E
13	☐ A	☒ B	☐ C	☐ D	☐ E
14	☐ A	☐ B	☐ C	☐ D	☒ E
15	☐ A	☒ B	☐ C	☐ D	☐ E
16	☐ A	☒ B	☐ C	☐ D	☐ E
17	☒ A	☐ B	☐ C	☐ D	☐ E
18	☐ A	☐ B	☐ C	☐ D	☒ E
19	☐ A	☐ B	☒ C	☐ D	☐ E
20	☒ A	☐ B	☐ C	☐ D	☐ E

ANULADA

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